SOLITARY WAVES IN MIXTURES OF LIQUID AND GAS BUBBLES

D P MASON

- SCHOOL OF COMPUTER SCIENCE AND APPLIED MATHEMATICS
- UNIVERSITY OF THE WITWATERSRAND

1. INDUSTRIAL PROCESSES WITH GAS BUBBLES

- GLASS FURNACE

  BUBBLES CAUSE FLAWS IN GLASS

- SUGAR CANE PROCESSING

  MIXTURE OF SHREDDED CANE AND JUICE

  REQUIRE BUBBLES TO REMOVE GASES

  NUCLEATION

- LAKE KIVU

  DEEP LAKE (474 m) BORDER BETWEEN RWANDA AND CONGO

  METHANE AND CARBON DIOXIDE DISSOLVED IN DEEP LAYERS

METHANE IS EXTRACTED TO GENERATE ELECTRICITY (25%)

GASES ENTER LAKE DUE TO VOLCANIC ACTIVITY

BUBBLES FORM WHEN

- LIQUID IS SUPERSATURATED (GAS ERUPTION)
- SEISMIC ACTIVITY (VOLCANIC ACTIVITY
  VERTICAL MOVEMENT OF GROUND)
- NUCLEATION SITES (DUST PARTICLES)

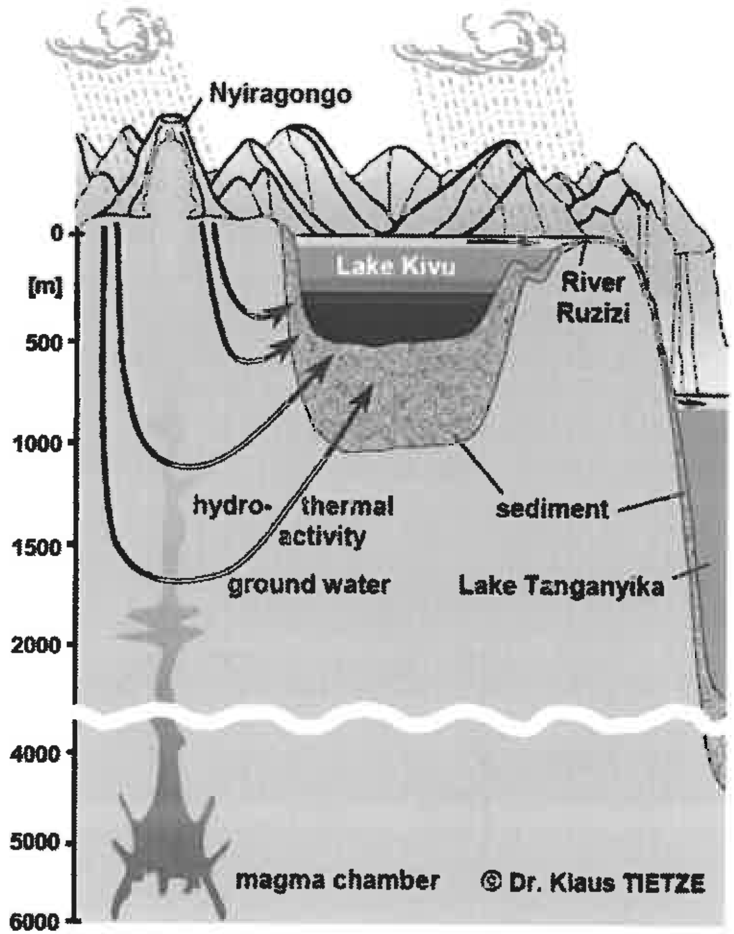


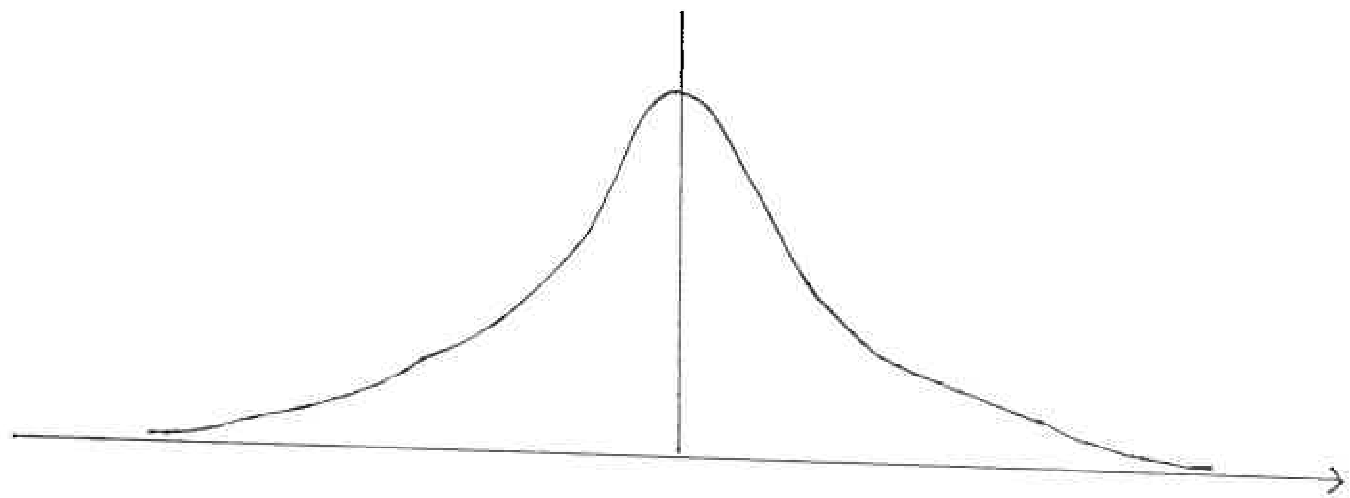
Figure 4 *Schematic cross section through Lake Kivu and Lake Tanganyika showing the mode of influx and infiltration of fresh and saline water into Lake Kivu (after Tietze, 2005).*

- PROPAGATION OF SOLITARY WAVES IN LIQUID BUBBLE MIXTURE

SOLITARY WAVE:

PULSE

RETAINS ITS SHAPE WHEN PROPAGATING OVER LARGE DISTANCE



SOLITON:

RETAIN THEIR SHAPE EVEN AFTER INTERACTING AMONG THEMSELVES

ACT LIKE PARTICLES

2 KORTEWEG - DE VRIES EQUATION

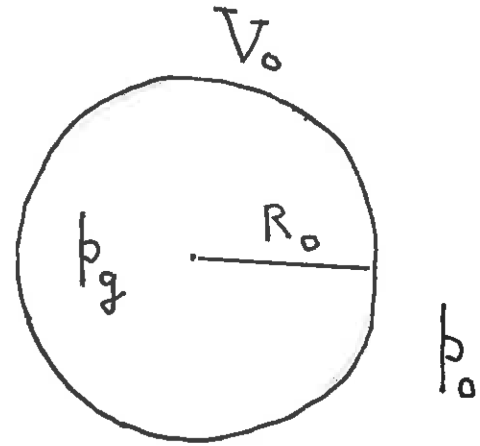
WE FIRST CONSIDER SOLUTIONS.

WE THEN LOOK AT MODELLING AND DERIVATION OF EQUATION.

DIMENSIONLESS FORM

$$\frac{\partial \xi}{\partial t} + (1 + \varepsilon \xi)\frac{\partial \xi}{\partial x} + \frac{\sigma}{6}\frac{\partial^3 \xi}{\partial x^3} = 0$$

NONLINEAR STEEPENING — DISPERSION BROADENING



PRESSURE PULSE

$$\frac{p_g}{p_0} = 1 + \varepsilon \xi(t, x) \qquad 0 < \varepsilon \ll 1$$

$$\sigma = \frac{R_0^3}{\lambda^2 (1 - n_0 V_0) n_0 V_0}$$

FIGURE 1. Diederik Korteweg (1848-1941) around 1898

FIGURE 2. Gustav de Vries (1866-1934)

n_0 = NUMBER OF BUBBLES PER UNIT VOLUME IN UNDISTURBED STATE

λ = CHARACTERISTIC WAVELENGTH

$$\frac{p_g}{p_0} = 1 + \varepsilon P(x - ct) \qquad W = x - ct$$

C = CONSTANT SPEED OF WAVE

BOUNDARY CONDITIONS

$$P(\pm\infty) = 0, \quad \frac{dP}{dW}(\pm\infty) = 0, \quad \frac{d^2P}{dW^2}(\pm\infty) = 0$$

$$\frac{p_g}{p_0} = 1 + a\,\mathrm{sech}^2\left[\left(\frac{a}{2\sigma}\right)^{\frac{1}{2}}(x - ct)\right]$$

$$C = 1 + \frac{a}{3}$$

$$\frac{p_g}{p_0} = 1 + a \operatorname{sech}^2\left[\left(\frac{a}{2\sigma}\right)^{\frac{1}{2}}(x - ct)\right], \quad c = 1 + \frac{a}{3}$$

SPEED OF SOLITARY WAVE DEPENDS ON AMPLITUDE a

LARGER AMPLITUDE WAVES TRAVEL FASTER

WE INVESTIGATE THE EFFECT OF BUBBLES ON THE PROPERTIES OF THE SOLITARY WAVE

REFERRED TO AS THE SINGLE SOLITON SOLUTION

3. TWO SOLITON SOLUTION (BARGMANN 1949)

RELATION BETWEEN SOLUTION $u(t,x)$ OF THE KdV EQUATION IN THE FORM

$$\frac{\partial u}{\partial t} - 6u\frac{\partial u}{\partial x} + \frac{\partial^3 u}{\partial x^3} = 0 \qquad (1)$$

AND THE SOLUTION $\psi(t,x)$ OF THE SCHRODINGER WAVE EQUATION

$$\frac{d^2\psi}{dx^2} + \left(k^2 - u(t,x)\right)\psi = 0 \qquad (2)$$

WITH POTENTIAL $u(t,x)$.

THE PROBLEM IS TO SOLVE FOR $u(t,x)$.

BARGMANN'S METHOD

LOOK FOR A SOLUTION OF THE FORM

$$\psi = \exp(ikx)\, F(k, x)$$

WHERE

$$F(k,x) = \text{POLYNOMIAL IN } k.$$

- LINEAR BARGMANN POTENTIAL (SINGLE SOLITON)

$$\psi_1 = \exp(ikx)\left[2k + i\,a(x)\right]$$

LEADS TO THE SINGLE SOLITON SOLUTION OF (1)

$$u(t,x) = -\frac{c}{2}\,\operatorname{sech}^2\left[\frac{\sqrt{c}}{2}(x-ct)\right]$$

- QUADRATIC BARGMANN POTENTIAL (INTERACTION TWO SOLITONS)

$$\psi_2 = \exp(ikx)\left[4k^2 + 2ik\,a(x) + b(x)\right]$$

LEADS TO THE TWO SOLITON SOLUTION OF (1)

$$u(t,x) = \frac{(p^2-q^2)\left[-2p^2 \operatorname{cosech}^2[p(x-4p^2t)] - 2q^2 \operatorname{sech}^2[q(x-4q^2t)]\right]}{\left[p \coth[p(x-4p^2t)] - q \tanh[q(x-4q^2t)]\right]^2}$$

WE WILL WRITE SOLUTION IN FORM FOR TWO SOLITARY WAVES IN A LIQUID BUBBLE MIXTURE.

WE WILL INVESTIGATE THE INTERACTION OF THE TWO SOLITARY WAVES. DO THEY RETAIN THEIR SHAPE AFTER INTERACTION. COMPUTER GRAPHS.

4. LIQUID BUBBLE MIXTURE MODEL

MODEL FOR A LIQUID BUBBLE MIXTURE

ASSUMPTIONS AND APPROXIMATIONS

SOME STEPS IN THE DERIVATION OF THE KdV EQUATION

- STUDY GROUP PROBLEM 4

  MOVEMENT OF BUBBLES IN A TUBE FOR METHANE EXTRACTION IN LAKE KIVU

REFERENCES

1. L VAN WIJNGAARDEN. ON THE EQUATIONS OF MOTION FOR MIXTURES OF LIQUID AND GAS BUBBLES. J FLUID MECH (1968), <u>33</u>, 465 - 474.

2. G L LAMB. ELEMENTS OF SOLITON THEORY, JOHN WILEY AND SONS, NEW YORK, 1980, CHAPTER 1.